Throughput Analysis and Design of Fixed and Adaptive ARQ/Diversity Systems For Slow Fading Channels

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Abstract

In this paper we study the performance of *Fixed Diversity (FD)* ARQ system in which no error correction coding is used, and only second order transmitter diversity is used. In the system two spatially separated antennas are used to transmit the same information signal in a staggerd fashion. In an FD system with two transmitting antennas. due to the staggered nature of transmssion, the information rate of the FD system is 1/2. and we compare this system with the block-coded hybrid ARQ system of rate 1/2 used in [1], [2], [3], [4]. Due to the lower information rate of the transmitter diversity system, we do not wish to diversify at all times, but only when diversity is required. Thus we propose an *Adaptive ARQ/Diversity (AD)* system, in which the transmitter adaptively switches between the single antenna and dual antenna mode using the implicit channel state information contained in the number of consecutive ACKs and NACKs. Using analytical techniques we show that the performance of the adaptive diversity system equals the performance of adaptive ARQ/FEC techniques that have been used in [5], [6], [7] which uses the uncoded and a rate half Maximum Distance Separable Code (MDS).

1 Introduction

In this paper, we propose to have several antennas at the transmitter end. Some restrictions on the spacing of the transmitting antennas must be heeded as documented in [8]. We assume that the antennas are spaced far enough apart so that each path seen from the transmitter to the receiver can be considered to be independent of each other. The requirements of the antenna spacing for 'decent' decorrelation of the antenna paths are available in [8].

The systems throughput is analyzed using a equivalent Markov characterization of the Rayleigh channel following the justifications of [1], [2], [9]. This Markov characterization of the Rayleigh channel has been utilized in by a general M-state markov Model in [2], and a specific 2 state model [10]. Here we use a 16 state model for the Rayleigh channel.

In this paper we incorporate the concept of transmitter antenna diversity in which the same inforation symbol is transmitted to the receiver during two consecutive symbol intervals in a staggered fashion from two spatially separated antennas. During the first symbol interval the information symbol is transmitted from one antenna, and in the next interval the same information is transmitted from the second antenna. This staggered transmission leads to the decrease in the information rate while achieving diversity gain. We propose the incorporation of such a transmitter diversity system with the basic non-adaptive ARQ protocols of [3]. We call this system the Fixed Diversity (FD) ARQ system.

We also propose an adaptive ARQ/Diversity (AD) system. Due to the penalty in the information rate that accompanies transmitter diversity, we would like to minimize the amount of time the transmitter needs to use the diversity system. In this system the transmitter uses either a single antenna transmission state or the dual diversity transmission state. We propose that the diversity system be used when the number of consecutive NACKs received by the transmitter surpasses a suitable threshold while the transmitter is in the single antenna state. After receiving a given number of consecutive ACKs while in the transmitter-diversity state the transmitter will revert to the single antenna state. This system has been entitled the AD system in this paper.

In Section 2 we describe the FD and AD systems. In Section 3. we analyse the FD and the AD systems. In section 4, we compare the performance of the AD and FD system with the performance of a hybrid $BCH(255,123)$ 19-bit error correcting coded system which has been studied in [1]. In Section 4. we also compare the AD system with the adaptive ARQ/FEC system which uses the BCH code state and the uncoded state as discussed in [3]. Also in that section we show that the performance of the AD system is similar to the performance of a adaptive ARQ/FEC system which uses an MDS code of rate 1/2 as the coded state. Finally in Section 5. we present our conclusions.

2 Description of Systems

The first system we discuss is called the Fixed Diversity (FD) system. This system uses basic non-adaptive ARQ protocols. In addition the transmitter uses a transmitter-diversity system without any error correction coding. The transmissions from each antenna is staggerd over two symbol intervals so that the receiver can separate the transmissions from each antenna at the receiver and obtain diversity gain. In this paper we limit the degree of the diversity to two. Due to the staggered transmission the information content of the packet is halved. Thus the FD system can transmit only half the information that can be transmitted when a single antenna is used, assuming no error correc-

tion coding. In the analysis, we assume that the receiver is able to decode the transmission from each antenna both individually, and jointly after maximal ratio combining. We assume further that the demodulation is coherent, and perfect estimates of the channel gains can be made for both antenna paths. There is some diversity gain that can be achieved at the cost of the loss in the information rate.

The AD system is similar to the adaptive ARQ/FEC protocols of [3]. However, in the AD system, the transmitter state is adaptively selected from the single antenna state and the transmitter diversity state, but there is no error correcting code used. This system is called the Adaptive ARQ/Diversity (AD) system. In this scheme, the two transmitter states are denoted as state '1' and state '2,3' and is shown in Figure 1. While in state '1' the transmitter uses only one antenna to transmit information. When α consecutive NACKs are received, the NACK counter being reinitialized at every received ACK, the transmitter switches to antenna 2 and 3, and transmits information using the two antennas in a staggered fashion over two symbol intervals. The staggered transmission leeds to the doubling of the information bit interval, halving the throughput. Once in the diversity state, the receiver counts the number of ACKs. The ACK counter is reset to zero whenever a NACK is received. When the number of consecutive ACKs exceeds the assigned threshold, γ, the transmitter switches back to the single antenna transmission state.

3 Analysis

In this section we analyse the Fixed Diversity (FD) and the Adaptive ARQ/Diversity (AD) systems that were described in Section 2.

Before we analyse the systems, we would like to restate the equations that describe the throughput of a basic ARQ system as were derived in [3]. The Markov parameters for the Rayleigh fading channel was set up in [1], [2], [9]. The same parameters are used in this paper. As a recapitulation, the steady state parameters for the 16 state Markov model, the states being labeled as $\{S_k\}_{k=1}^{16}$ in order of increasing instantaneous SNR, is given by $\vec{\pi}$ and the one step transition probability matrix is given by $\vec{P}$. For the analysis of a GBN protocol where the round-trip delay is d frames long, we need the d-step transition probability matrix denoted as $\vec{T} = \vec{P}^d$. The matrix $\vec{G}$ is defined such that $G_{i,j} = T_{j,i} Pr(\mathcal{F}|S_i)$, where for a t-error correcting code, with n bits per frame using BPSK modulation:

$$Pr(\mathcal{F}|S_k) = \sum_{j=t+1}^{n} \binom{n}{j} \left(\frac{1}{2} erfc\left(\sqrt{l_k}\right)\right)^{j} \times \left(1 - \frac{1}{2} erfc(\sqrt{l_k})\right)^{n-j} \quad (1)$$

is the probability of failure of a frame given the channel state is S_k and $\mathcal{F}$ denotes failure. Here, l_k are the quantization levels found in [1]. We also use

$$\vec{\Gamma} = [\pi_1 Pr(\mathcal{F}|S_1)\ \pi_2 Pr(\mathcal{F}|S_2)\ \cdots\ \pi_k Pr(\mathcal{F}|S_k)]',$$

which is a column vector whose ith element denotes the probability of the joint event of the channel initially being at state S_i and failure resulting given that state.

When the frames are encoded using a rate r FEC, we found that the throughput can be represented as:

$$T_r = \frac{1}{N_r} \quad (2)$$

where,

$$N_r = \frac{1}{r} + \frac{d}{r} \vec{1} \sum_{i=0}^{\infty} \vec{G}^i \vec{\Gamma} = \frac{1}{r} + \frac{d}{r} \vec{1} \left[\vec{I} - \vec{G}\right]^{-1} \vec{\Gamma}.$$

Here, $\vec{1} = [11 \cdots 1]$ is a (1×16) row-vector, and $\vec{I}$ is a (16×16) dimensional identity matrix. In [1], it has been shown that the eigen-values of the $\vec{G}$ matrix are all less than unity, and consequently, the above series converges. In [1] it was stated that if r is less than 1 then the system is called a hybrid ARQ system. If the rate is 1, then it is a basic uncoded ARQ system.

3.1 Fixed Diversity (FD)

When the same information is transmitted to the receiver from two transmitting antennas in a staggerd fashion the receiver can decode the transmission from each antenna and test the received vector with the CRC code. If the individual transmissions fail the CRC check, then the transmissions can be combined coherently and then the CRC checking can be performed once more. This way we can reduce the probability of failure of the frame transmissions. In this section we only use a dual diversity system. Due to the staggerd transmission of the two transmitting antennas the information rate is halved and we need to take it into consideration in evaluating the throughput. The packet fails if both transmissions from each antenna fail the CRC, and the maximal-ratio diversity combined frame also yields a CRC error. For the frame to be successfully decoded any one of the transmissions, or the combined signal has to pass the CRC check. The system uses two antennas labeled '2' and '3', and it is assumed that the channels seen by the antennas are independent of one another.

The conditional probability of a failure given that the channel state for antenna '2' and antenna '3' are S_i and S_j respectively is given to be (assuming ideal coherent BPSK modulation and perfect maximal ratio combining):

$$\begin{aligned} & Pr(\mathcal{F}_{2,3}|s^{(2)} = S_i, s^{(3)} = S_j) \\ &= \left\{1 - \left(1 - \tfrac{1}{2} erfc(\sqrt{l_i})\right)^{n/2}\right\} \\ &\times \left\{1 - \left(1 - \tfrac{1}{2} erfc(\sqrt{l_j})\right)^{n/2}\right\} \\ &\times \left\{1 - \left(1 - \tfrac{1}{2} erfc(\sqrt{(l_i + l_j)}\right)^{n/2}\right\}. \end{aligned} \quad (3)$$

Which states that the frame, now containing only $n/2$ bits fails to pass the CRC check individually and after diversity combining. For multichannel detection, we use the result of [11] to evaluate the bit error probability of the combined signal, which simply states that the SNRs of the two channels add when the estimates of the individual channels are perfect.

The probability of the frame being accepted at the receiver is then given to be:

$$
\begin{aligned}
&Pr(\mathcal{S}_{2,3}|s^{(2)} = S_i, s^{(3)} = S_j) \\
&= 1 - Pr(\mathcal{F}_{2,3}|s^{(2)} = S_i, s^{(3)} = S_j) \qquad (4)
\end{aligned}
$$

We define the $\vec{G}_{(2,3)}$ matrix, where the subscripts denote the antenna numbers, such that $(\vec{G}_{(2,3)})_{i_1 j_1 i_2 j_2}$ represents the joint probability of the channel of antenna '2' transitioning from state S_{i_2} to S_{i_1} in d steps, the channel of antenna '3' transitioning from state S_{j_2} to S_{j_1} in d steps and the frames received over the channels being in error both individually and after diversity combining. The matrix is 256×256, reflecting the joint states of antennas '2' and '3' where each channel is represented as a 16 state Markov model. Each element of $\vec{G}_{(2,3)}$ is given as:

$$
\begin{aligned}
&\left\{(\vec{G}_{(2,3)})_{i,j}\right\}_{i,j=1}^{256} \\
&= \left\{(\vec{G}_{(2,3)})_{16(i_1-1)+j_1,\,16(i_2-1)+j_2}\right\}_{i_1,i_2,j_1,j_2=1}^{16} \qquad (5) \\
&= \left\{(\vec{G}_{(2,3)})_{i_1 j_1 i_2 j_2}\right\}_{i_1,i_2,j_1,j_2=1}^{16}
\end{aligned}
$$

Thus the rows and columns of $\vec{G}_{(2,3)}$ are represented by the 2-tuples denoted by (i_1, j_1) and (i_2, j_2). The indices i_1 and i_2 represents the d-step transition of the channel of antenna '2' from state S_{i_2} to state S_{i_1}, and j_1 and j_2 represents the d-step transition of the channel of antenna '3' from state S_{j_2} to state S_{j_1}.

Now we define the matrix $\vec{G}_{(2,3)}$ such that:

$$
\begin{aligned}
&(\vec{G}_{(2,3)})_{i_1 j_1 i_2 j_2} \\
&= \mathcal{T}_{i_2 i_1} \mathcal{T}_{j_2 j_1} Pr(\mathcal{F}_{2,3}|s^{(2)} = S_{i_1}, s^{(3)} = S_{j_1}). \qquad (6)
\end{aligned}
$$

Where $\mathcal{T}$ denotes the d step transition probability matrix of each channel.

In a similar manner the 256×1 column vector $\vec{\Gamma}_{(2,3)}$ is defined such that:

$$
\begin{aligned}
\left\{(\vec{\Gamma}_{(2,3)})_i\right\}_{i=1}^{256} &= \left\{(\vec{\Gamma}_{(2,3)})_{16(i_1-1)+j_1}\right\}_{i_1,j_1=1}^{16} \qquad (7) \\
&= \left\{(\vec{\Gamma}_{(2,3)})_{i_1 j_1}\right\}_{i_1,j_1=1}^{16}
\end{aligned}
$$

and is given by:

$$
(\vec{\Gamma}_{(2,3)})_{i_1 j_1} = \pi_{i_1} \pi_{j_1} Pr(\mathcal{F}_{2,3}|s^{(2)} = S_{i_1}, s^{(3)} = S_{j_1}). \qquad (8)
$$

This represents the joint proability of the two channels initially being in state S_{i_1} and S_{j_1} and the frames received over the channels being in error both individually and after diversity combining. The matrix $\vec{H}_{(2,3)}$ analogous to $\vec{G}_{(2,3)}$ but dealing with the probabilities of successful frame transmission is defined such that:

$$
\begin{aligned}
(\vec{H}_{(2,3)})_{i_1 j_1 i_2 j_2} &= \mathcal{T}_{i_2 i_1} \mathcal{T}_{j_2 j_1} \\
&\times (1 - Pr(\mathcal{F}_{2,3}|s^{(2)} = S_{i_1}, s^{(3)} = S_{j_1})). \qquad (9)
\end{aligned}
$$

The vector $\vec{L}_{(2,3)}$, analogous to $\vec{\Gamma}_{(2,3)}$ is given as:

$$
\begin{aligned}
(\vec{L}_{(2,3)})_{i_1 j_1} &= \pi_{i_1} \pi_{j_1} \\
&\times (1 - Pr(\mathcal{F}_{2,3}|s^{(2)} = S_{i_1}, s^{(3)} = S_{j_1})). \qquad (10)
\end{aligned}
$$

The average number of bits transmitted for each correctly received bit using the FD system is given by, following the same procedure as was used to derive (2) which was developed in [1]:

$$
\begin{aligned}
N^{FD} &= 2 + 2d\vec{1} \sum_{i=0}^{\infty} \vec{G}^{i}_{(2,3)} \vec{\Gamma}_{(2,3)} \qquad (11) \\
&= 2\left[1 + d\vec{1}_{div}\left[\vec{I}_{div} - \vec{G}_{(2,3)}\right]^{-1} \vec{\Gamma}_{(2,3)}\right].
\end{aligned}
$$

Here $\vec{1}_{div}$ is a 1×256 row vector of all ones, and $\vec{I}_{div}$ is the 256×256 identity matrix. The throughput of the FD system is then simply given as:

$$
T_{FD} = \frac{1}{N^{FD}}. \qquad (12)
$$

In this sub-section we have set up most of the equations necessary to evaluate the performance of the Adaptive ARQ/Diversity (AD) system. In the next sub-section we analyse the AD system.

3.2 Adaptive ARQ/Diversity (AD)

In light of the reduced maximum achievable throughput due to diversity transmission, we would like to limit the use of the diversity transmission to the times when its use becomes necessary. In the AD system the diversity transmission mode is switched on adaptively. The system is described in Figure 1. The system initially uses antenna '1'. When α NACKs are received while using antenna '1', the system switches to two different antennas '2' and '3', and uses the dual diversity system. It is assumed that the channels seen by all the antennas are independent of one another.

In our adaptive ARQ/Diversity system the transmitter transitions from the single antenna state to the diversity state when it receives α consecutive NACKs. The probability of transitioning from the single antenna to the dual-antenna transmission system is given by:

$$
\Upsilon_{1,(2,3)} = \vec{1}\vec{G}^{\alpha-1}_{(1)} \vec{\Gamma}_{(1)} \qquad (13)
$$

Where $\vec{G}_{(1)}$ and $\vec{\Gamma}_{(1)}$ are the same as the matrices $\vec{G}$ and $\vec{H}$ that were found for a code rate of 1.0 in [1]. The subscripts of $\vec{G}$ and $\vec{\Gamma}$ in (13) denotes that antenna '1' is being used in the single antenna transmission mode. The matrix $\vec{G}_{(1)}$ is given to be:

$$
\vec{G}_{(1)} =
\begin{bmatrix}
\mathcal{T}_{1,1} Pr(\mathcal{F}|S_1) & \mathcal{T}_{2,1} Pr(\mathcal{F}|S_1) & \cdots & \mathcal{T}_{M,1} Pr(\mathcal{F}|S_1) \\
\mathcal{T}_{1,2} Pr(\mathcal{F}|S_2) & \mathcal{T}_{2,2} Pr(\mathcal{F}|S_2) & \cdots & \mathcal{T}_{M,2} Pr(\mathcal{F}|S_2) \\
\vdots & \vdots & \ddots & \vdots \\
\mathcal{T}_{1,M} Pr(\mathcal{F}|S_M) & \mathcal{T}_{2,M} Pr(\mathcal{F}|S_M) & \cdots & \mathcal{T}_{M,M} Pr(\mathcal{F}|S_M)
\end{bmatrix}
$$

and $\vec{\Gamma}_{(1)}$ is:

$$
\vec{\Gamma}_{(1)} = [\pi_1 Pr(\mathcal{F}|S_1)\ \pi_2 Pr(\mathcal{F}|S_2) \cdots \pi_k Pr(\mathcal{F}|S_k)]'.
$$

Where,

$$
Pr(\mathcal{F}|S_k) = 1 - \left(1 - \frac{1}{2} erfc(\sqrt{l_k})\right)^n, \qquad (14)
$$

is the conditional probability of the uncoded n bit long frame being in error given that the frame is transmitted

during channel state S_k, and $\vec{T}$ denotes the d step transition probability matrix of the channel as found in [1].

Since the transmitter uses the diversity mode until γ consecutive ACKs are received, the probability of going from the diversity state to the single antenna transmission state is given by:

$$\Upsilon_{(2,3),1} = \vec{1}_{div} \bar{H}_{(2,3)}^{-1} \bar{L}_{(2,3)} \tag{15}$$

where, $\vec{1}_{div}$ is the 1×256 row vector of all ones. The matrix $\bar{H}_{(2,3)}$ and the vector $L_{(2,3)}$ has been derived in sub-section 3.1.

From the transition probabilities found in (13), and (15), the steady state probability of the transmitter staying in the single antenna mode is given by:

$$\Phi_1 = \frac{\Upsilon_{(2,3),1}}{\Upsilon_{1,(2,3)} + \Upsilon_{(2,3),1}} \tag{16}$$

and, the steady state probability of the transmitter staying in the diversity mode is given by:

$$\Phi_{2,3} = \frac{\Upsilon_{1,(2,3)}}{\Upsilon_{1,(2,3)} + \Upsilon_{(2,3),1}} \tag{17}$$

The average number of transmitted bits per correctly received bit, given that the system is in the single antenna mode is given by:

$$\begin{aligned} N_{(1)} &= 1 + d\vec{1}\left[\bar{I} - \vec{G}^{o}_{(1)}\right]\left[\bar{I} - \vec{G}_{(1)}\right]^{-1} \vec{V}_{(1)} \\ &+ 2d\left\{\vec{1}\vec{G}^{o}_{(1)}\vec{T}^{1}\vec{V}_{(1)}\right\} \vec{1}_{div}\left[\bar{I}_{div} - \vec{G}_{(2,3)}\right]^{-1} \vec{V}_{(2,3)} \end{aligned} \tag{18}$$

Here, $\bar{I}_{div}$ is a 256×256 identity matrix.

The average number of transmitted bits per correctly received bit, given that the system is in the diversity mode, is given by:

$$N_{(2,3)} = 2\left[1 + d\vec{1}_{div}\left[\bar{I}_{div} - \vec{G}_{(2,3)}\right]^{-1} \vec{V}_{(2,3)}\right] \tag{19}$$

The throughput of the adaptive ARQ/Diversity system is:

$$T_{AD} = \Phi_{(1)} \frac{1}{N_{(1)}} + \Phi_{(2,3)} \frac{1}{N_{(2,3)}} \tag{20}$$

We have presented the analytical results for the throughput of the the FD and the AD systems that have been described in Section 2. We present our discussion of the analytical results in Section 4.

4 Discussion of Results

In Section 2 the FD and AD systems were described. In Section 3 the analysis of the throughput of the systems was presented. In this section we discuss the performance of these schemes.

In this section we also consider the performance of ARQ systems where the FEC is chosen to be the 19-error correcting $BCH(255,123)$ code [12]. The frame size for this system is set to 255 bits. The rate of this code is 123/255, and the frame error probability for this code given that the channel in in state S_k is given to be:

$$Pr(\mathcal{F}|S_k) = \sum_{i=19+1}^{n} \binom{n}{i} \left(\tfrac{1}{2}erfc\left(\sqrt{l_k}\right)\right)^{i} \times \left(1 - \tfrac{1}{2}erfc(\sqrt{l_k})\right)^{n-i} \tag{21}$$

Here, we assume that the BCH code can correct any pattern of errors if the number of errors received in the frame is at most 19 bits. Also, $\mathcal{F}$ denotes failure, and l_k and S_k are the kth quantization level and kth state as found in [1]. We choose the BCH code because we would like to compare the FD system with a realistic code of similar rate as the FD system. The maximum distance separable (MDS) codes of [1] and [3], are ideal codes and no binary MDS codes have been found [11].

4.1 FD and AD Systems

In this sub-section we first compare the performance of the hybrid-ARQ protocols with the FD system. For the FD system the number of bits per frame is set at 256 bits, with a bit-rate of 192 Kbits/sec. This gives a frame rate of 750 frames per second. For all the analytical results presented here, the carrier is at 900 MHz. The modulation is considered to be BPSK with ideal coherent demodulation. Since the FD system accommodates only half the information bits, the effective information rate for the system is 1/2, similar to the rate 1/2 $BCH(255,123)$ code. Therefore, we can compare the hybrid (BCH) system to the diversity system.

The performance of the FD system, for a round-trip delay of 2 frames is shown in Figure 2 for channel Doppler rates of 5/6, 0.10, and 10.0 Hz. The figure also shows the corresponding performance that is obtained by using the hybrid $BCH(123,255)$ coded system. We see that at the lower SNRs the performance of the FD system is superior to the hybrid BCH system. The performance of the FD system is about 8 dB better than the hybrid (BCH)system at a throughput of 30%. The performance of both systems improve as the Doppler rate increases. The difference in performance at 10 Hz Doppler and at the 30% throughput level is about 4 dB. At 0 dB SNR the FDsystem yields a throughput of 15% at a Doppler rate of 5/6 Hz, whereas the hybrid BCH system yields a throughput of 3% only.

For the AD system, the number of bits per frame is set at 256 bits, with a bit-rate of 192 Kbits/sec. This gives a frame rate of 750 frames per second. Again the round-trip delay is 2 frame durations. Throughout this section we consider that the diversity system is initiated when the transmitter receives 2 NACKs while in the single antenna mode. If two consecutive ACKs are received in the diversity state the transmitter reverts to the single antenna state. We compare the AD system with the adaptive ARQ/FEC system of [3], where the transmitter adaptively selects it code state from the uncoded and rate 1/2 $BCH(255,123)$ coded states. The adaptive ARQ/FEC (BCH) system also switches from the uncoded state to the coded state when 2 consecutive NACKs are received and returns to the uncoded state when 2 consecutive ACKs are received. Since the dual diversity mode accommodates only half the information bits, the effective information rate for the diversity system is 1/2, similar to the rate 1/2 $BCH(255,123)$ code.

Therefore, we can compare the adaptive ARQ/FEC (BCH) system to the AD system.

In Figure 3, we plot the performance of the basic uncoded system, the hybrid BCH system, the FD system, the adaptive ARQ/FEC (BCH) system and the AD system at a Doppler rate of 5/6 Hz. The performance of the AD system is similar to the adaptive ARQ/FEC (BCH) system when the average SNR is above 17 dB. Below this SNR the AD system performs much better than the adaptive ARQ/FEC (BCH) system. At 50% throughput the AD system gives 4 dB better performance than the adaptive ARQ/FEC (BCH) system.

In Figure 4, the performance of the systems are shown for a Doppler rate of 0.1 Hz. In this case the difference in performance for the adaptive ARQ/FEC (BCH) system and the AD system is more marked. The performance of the two systems are similar above 18 dB average SNR. The AD system is about 8 dB superior to the adaptive ARQ/FEC (BCH) system at the 50% throughput level. The throughput of the systems at a Doppler rate of 10 Hz is shown in Figure 5. The performance difference of the two systems is smaller at a higher Doppler rate.

We have compared the FD system with the adaptive ARQ/FEC (BCH) system. Now we take a look at the performance that can be obtained if an (MDS) code were available. In Figure 6, we plot the performance of the basic uncoded, the rate 1/2 hybrid (MDS) system, the FD, the adaptive ARQ/FEC (MDS) and the AD system for a Doppler of 5/6 Hz and 256 bits per frame. The frame rate is 750 frames/sec. The adaptive ARQ/FEC (MDS) system switches from the uncoded state to the coded state when 2 consecutive NACKs are received and switches back to the uncoded state from the coded state when 2 consecutive ACKs are received. From Figure 6 we see that the performance of the FD system is almost the same as the hybrid (MDS) system. When the average SNR is above 4 dB the FD system performs marginally better than the hybrid (MDS) system. The adaptive ARQ/FEC (MDS) and the AD systems are also very similar in performance. The AD system is marginally better than the adaptive ARQ/FEC (MDS) system when the SNR is between 3 and 13 dB. Since the performance of the hybrid (MDS) system is an upper-bound of the performance that can be obtained with a real block code of similar rate, we can conclude that when the fading rate is low, at almost all SNRs the FD system will most likely out-perform the rate 1/2 block-coded hybrid ARQ system. Also the AD system will most likely out-perform the adaptive ARQ/FEC systems which uses the uncoded and rate 1/2 block-coded states.

5 Conclusion

In this paper we have proposed the Fixed Diversity (FD) and the Adaptive ARQ/Diversity (AD) systems. We have analysed the performance of the FD and AD systems. In the FD system the transmitter always uses dual transmitter diversity. In the AD system the transmitter adaptively switches between the single antenna transmission mode and the dual transmitter diversity mode. We have observed that in slow fading channels the performance of the adaptive ARQ/FEC (BCH) system is quite inferior to the AD system. We have selected a good block code for the adaptive ARQ/FEC system and the AD system performance has been found to exceed the adaptive ARQ/FEC (BCH) system's performance by as much as 7 to 8 dB. We have also noted that the performance of the AD system is comparable to the performance of the adaptive ARQ/FEC (MDS) system.

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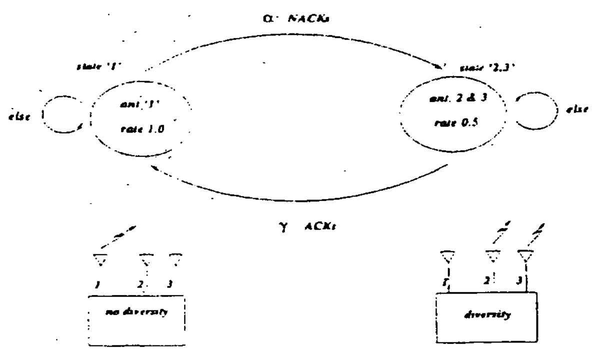

Figure 1: The Adaptive ARQ/Diversity (AD) System.

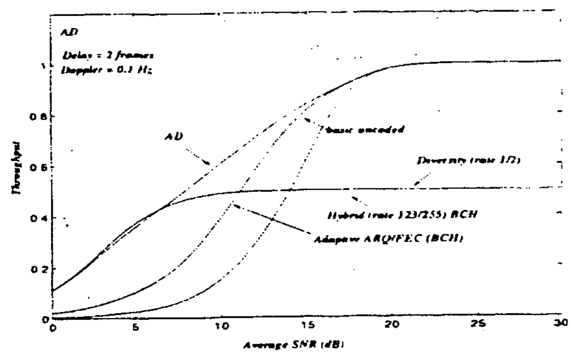

Figure 4: Throughputs for AD and Adaptive ARQ/FEC (BCH) (Doppler = 0.01 Hz).

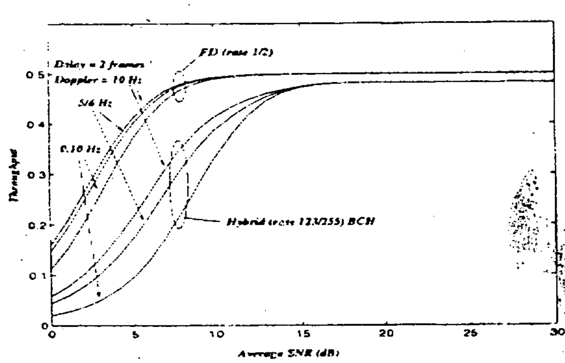

Figure 2: Throughputs for FD and Hybrid (BCH) System.

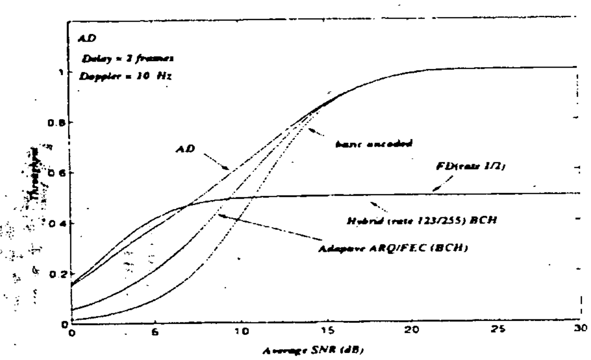

Figure 5: Throughputs for AD and Adaptive ARQ/FEC (BCH) (Doppler = 10.0 Hz).

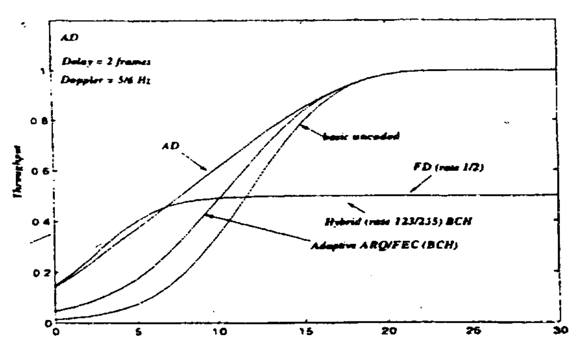

Figure 3: Throughputs for AD and Adaptive ARQ/FEC (BCH) (Doppler = 5/6 Hz).

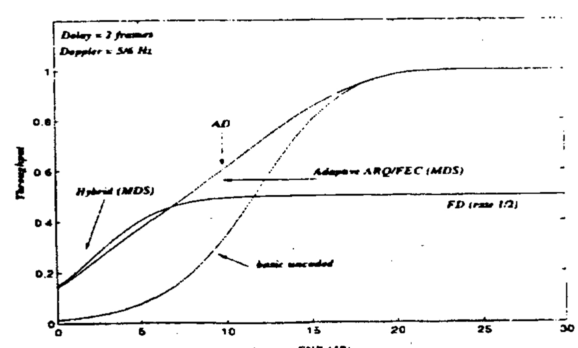

Figure 6: Throughputs for AD and Adaptive ARQ/FEC(MDS) Systems.